Accuracy of mass and radius determination for neutron stars from simulated LOFT spectra

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Plan of my talk

- hot neutron stars model atmospheres (ATM code)
  - basic assumptions
  - principal equations
  - Compton redistribution function
- preparing a spectrum as seen by LOFT satellite
- fitting procedure
- accuracy of mass and radius determination for a neutron star
- summary and plans for the future

Assumptions of our model atmospheres

- plane-parallel geometry
- hydrostatic and radiative equilibrium
- equation of state of gas: local thermodynamic equilibrium
- non-rotating neutron star
- magnetic field does not modify opacity coefficients
- we do not include relativistic corrections in model atmosphere equations
- we include f-f, b-f, b-b processes and Compton scattering
- photons are scattered on relativistic electrons with thermal velocity distribution
- we allow for large relative energy and momentum exchange between photon and electron during a single scattering
- we reject well-known Kompaneets approximation!

Principal equations

1. Equation of radiative equilibrium

At each point in the atmosphere total energy absorbed by given volume of gas should be equal to the energy emitted by this gas.

$$\int_0^\infty (\kappa_\nu + \sigma_\nu) J_\nu \, d\nu = \int_0^\infty (\kappa_\nu + \sigma_\nu) S_\nu \, d\nu$$

energy absorbed   energy emitted

2. Equation of hydrostatic equilibrium

Gradients of gas and radiation pressures are balanced by the gravitational acceleration.

$$\frac{dP_{gas}}{dz} + \frac{dP_{rad}}{dz} = -\rho g$$

Atmosphere is static - do not expand or move downward.

Principal equations

3. Radiative transfer equation

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - \epsilon_\nu B_\nu - (1-\epsilon_\nu) J_\nu + (1-\epsilon_\nu)(J_\nu - B_\nu) \int_0^\infty \Phi_1(\nu,\nu')\, d\nu' - (1-\epsilon_\nu) \int_0^\infty (J_{\nu'} - B_{\nu'}) \Phi_2(\nu,\nu')\, d\nu'$$

$$\Phi_1(\nu,\nu') = \left(1 + \frac{c^2}{2h\nu'^3}\right) \Phi(\nu,\nu')$$

$$\Phi_2(\nu,\nu') = \left(1 + \frac{c^2}{2h\nu'^3}\right) \left(\frac{\nu}{\nu'}\right)^3 \exp\left[-\frac{h(\nu-\nu')}{kT}\right] \Phi(\nu,\nu')$$

$\Phi(\nu,\nu')$ - Compton redistribution function normalized to unity.

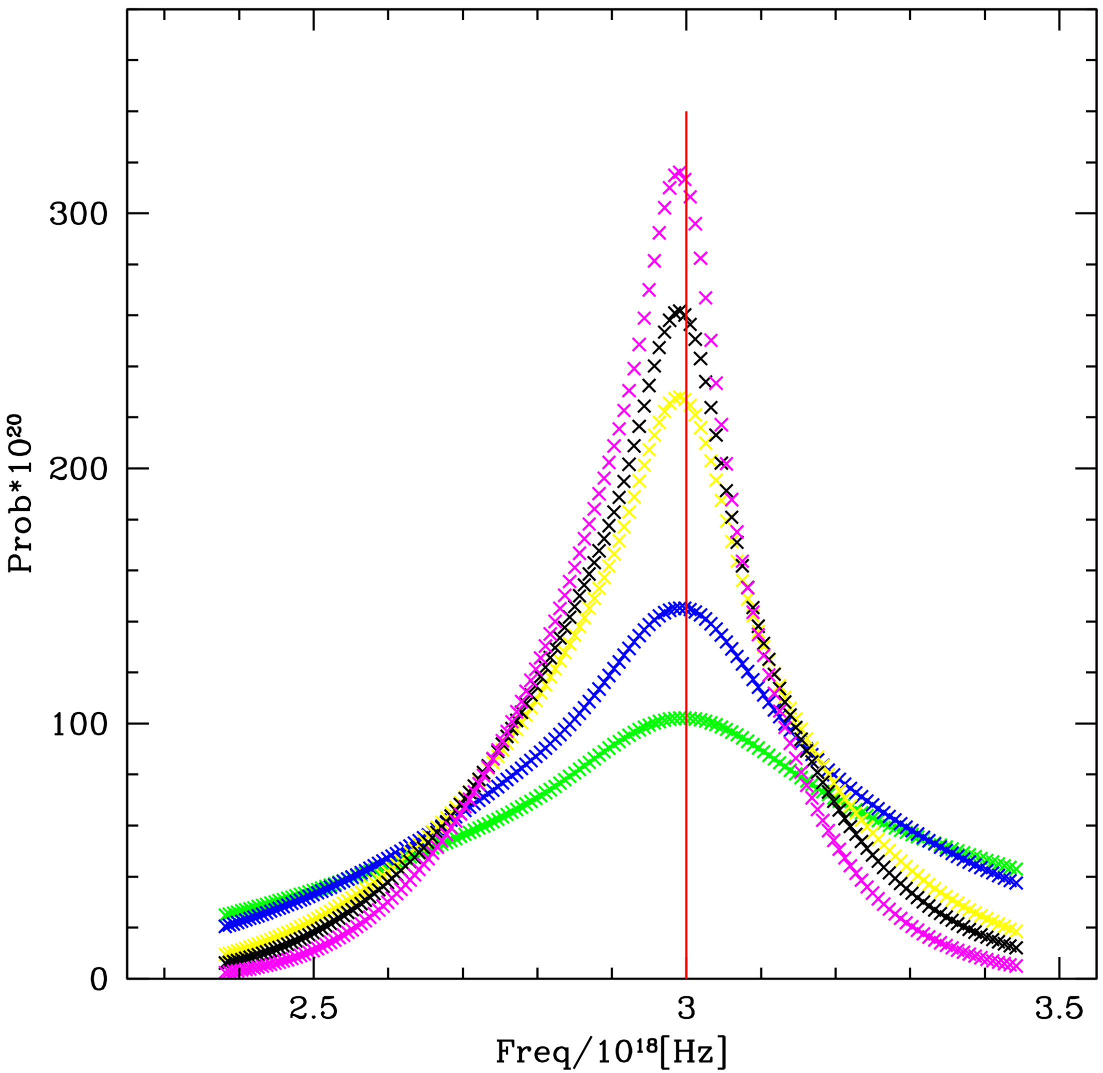
Photon redistribution function

We calculated photon redistribution function Φ(ν,ν') following Guilbert (1981), **BUT** we corrected his formulae.

Klein-Nishina corrections were included here.

Rescaled probability of scattering a photon of initial wavelength λ = 1 Å and for electron temperatures (in 10^6 K):

magenta	10
black	15
yellow	20
blue	50
green	100



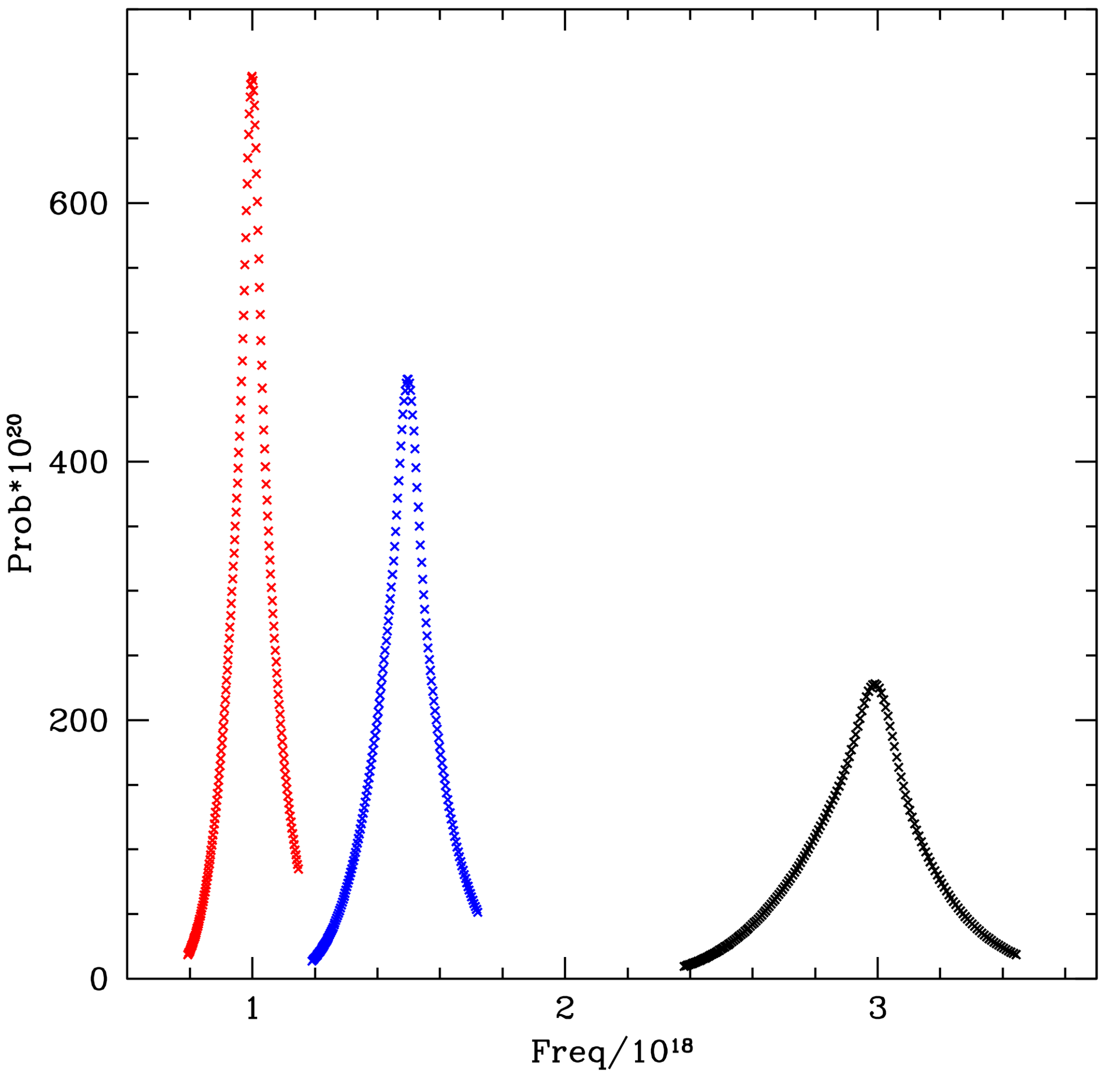
Photon redistribution function

We calculated photon redistribution function Φ(ν,ν') following Guilbert (1981), **BUT** we corrected his formulae.

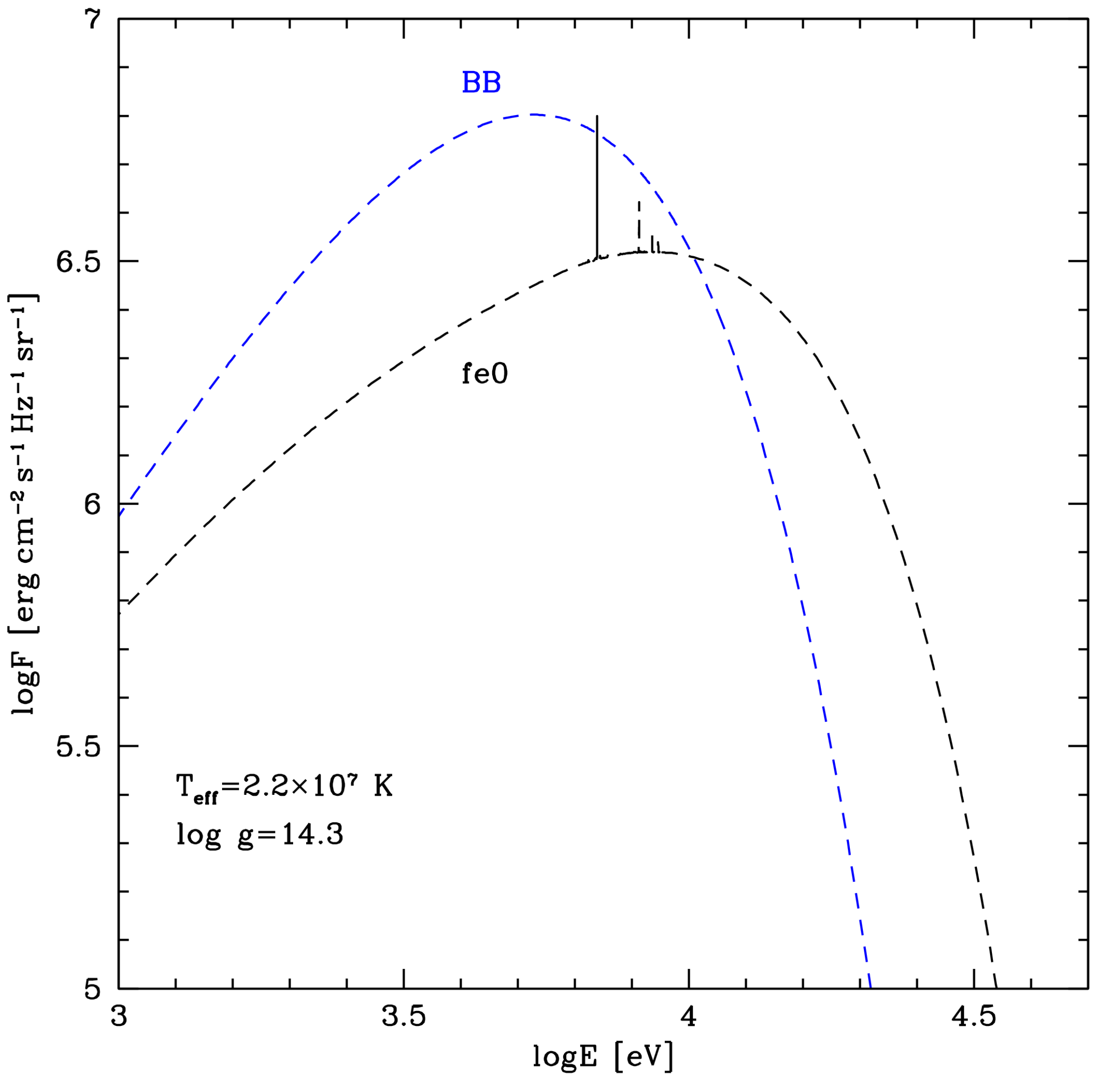
Klein-Nishina corrections were included here.

Rescaled probability of scattering a photon for electron temperature T = 2·10^7 K and initial photon wavelengths:

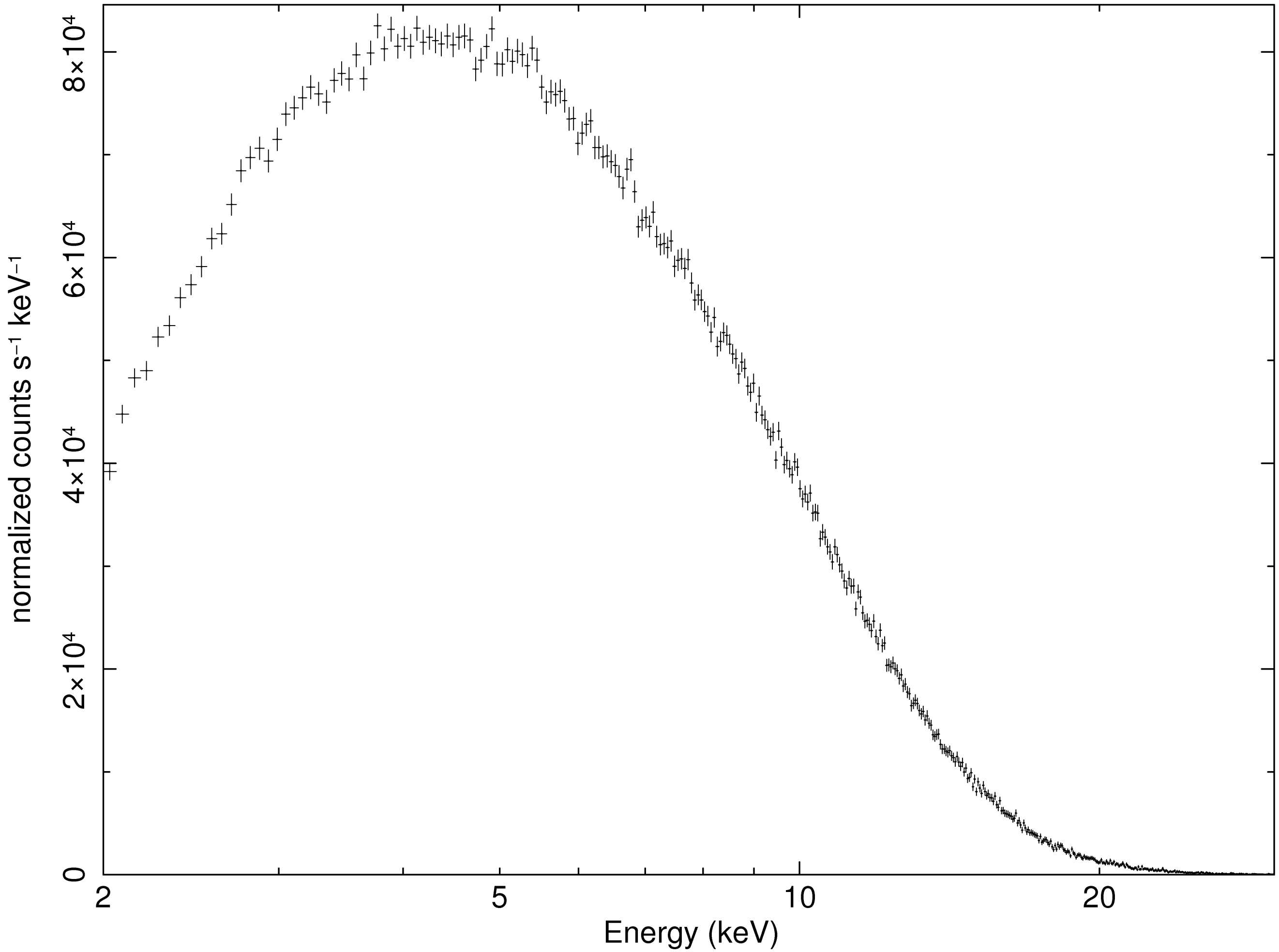
red	λ = 3 Å
blue	λ = 2 Å
black	λ = 1 Å



Model atmosphere computations



Synthetic spectrum



Our numerical spectrum after convolving it with the LAD response matrix.

spectrum was integrated over 1 sec

log(g), z, T_{eff} and chemical composition of the atmosphere was assumed.

Fitting procedure

- we calculated 227 models for the chemical composition: mass abundances X = 0.6950, Y = 0.3036 and Z = $1.425 \cdot 10^{-3}$ (only Fe)
- for each combination of T_{eff}, log(g) and z: we determined only a normalization factor N_{ATM} corresponding to the minimum value of χ^2
- then, we ignored N_{ATM} and took into account only χ^2_{min} as the function of T_{eff}, log(g) and z
- we find the minimum χ^2 in 3-dimensional space of parameters
- effective temperature, corresponding to the minimum χ^2 is not essential in further considerations
- log(g) and z, corresponding to the minimum χ^2, determine the best value of mass M and radius R of the compact star

Mass and radius determinations

Mass and radius of the compact star was determined from:

$$R = \frac{zc^2}{2g} \frac{(2+z)}{(1+z)}$$

$$M = \frac{z^2 c^4}{4gG} \frac{(2+z)^2}{(1+z)^3}$$

Majczyna & Madej (2005)

Accuracy of log(g) and z determination for a neutron star

- 3σ confidence error of log(g) and z was determined from the condition: $\chi_{min}^2 < \chi^2 < \chi^2_{min} + \Delta\chi^2$ where $\Delta\chi^2 = 9$ (for 1 free parameter - N_{ATM} and for all degrees of freedom)
- we defined errors of log(g) and z as a width of 3σ "confidence level contour" for the gravity and redshift parameters

$$\boldsymbol{\delta \log(g) \approx 0.1} \text{ (cgs units)}$$
$$\boldsymbol{\delta z \approx 0.03}$$

- using these values we determined mass and radius errors
- for *z,* confidence level is higher than the step (0.01) in fitting procedure but for log(g) is equal. So, theoretical models are calculated with too high step in log(g) parameter
- too high step in log(g) parameter causes that 3σ confidence contour contains only 2 points ⇒ we need more dense grid of models

Accuracy of mass and radius determination for a neutron star

$$\delta M = \frac{\partial M}{\partial g}\delta g + \frac{\partial M}{\partial z}\delta z$$

$$\delta R = \frac{\partial R}{\partial g}\delta g + \frac{\partial R}{\partial z}\delta z$$

$$\delta M = \frac{c^4}{4Gg}\frac{z(z^3 + 4z^2 + 8z + 8)}{(z+1)^4}\delta z - \frac{c^4}{4Gg^2}\frac{z^2(z+2)^2}{(z+1)^3}\delta g$$

$$\delta R = \frac{c^2}{2g}\left(2 - \frac{z(z+2)}{(z+1)^2}\right)\delta z - \frac{c^2}{2g^2}\frac{z(z+2)}{(z+1)}\delta g$$

3σ mass and radius errors:

$\delta M = 0.12\ M_\odot$
$\delta R = 1.68$ km

Summary

- we fitted extensive grid (227 models) of theoretical models of hot neutron star atmospheres to the single synthetic LAD spectrum
- low theoretical observational errors of the fitting spectrum allowed us to very accurate determination of mass and radius (previously assumed) of the neutron star
- errors of log(g) and z were defined as 3σ contours are equal to $\delta\log(g) \approx 0.1$ and $\delta z \approx 0.03$
- mass and radius errors were calculated from $\delta\log(g)$ and δz:

$$\boldsymbol{\delta M = 0.12\ M_\odot}$$
$$\boldsymbol{\delta R = 1.68\ \mathrm{km}}$$

- we expect that the above errors could be lower when we will fit new more numerous grid ($\Delta\log(g) = 0.02$, $\Delta T_{eff} = 2 \cdot 10^6$ K) of our numerical models
- calculations are in progress: within 2 to 3 months new grid of models should be obtained for the same chemical composition

Thank you

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