The equation of state problem

effective theories and first principles

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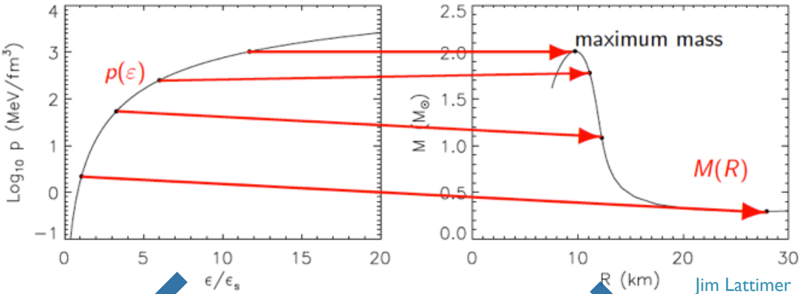
UK LOFT meeting, 24 June 2013

Equation of State

Tolman-Oppenheimer-Volkov equations

$$\frac{dP}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi P r^3)(\epsilon + P)}{r(r - 2Gm/c^2)} \qquad \frac{dm}{dr} = \frac{4\pi\epsilon r^2}{c^2}$$

Equation of State — Mass-Radius relation



Jim Lattimer

Nuclear quantity
(that's what I do!)

Stellar quantity

Equation of State

Tolman-Oppenheimer-Volkov equations

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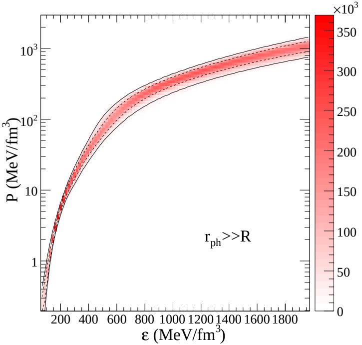
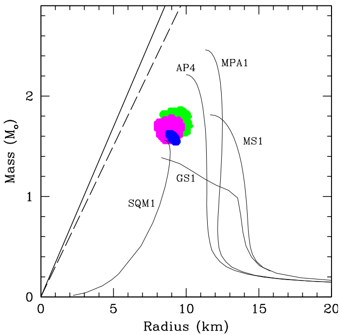
Empirical EoS



Steiner *et al.*, ApJ 722, 33 (2010)

Mass-radius relation



Özel, Baym & Güver, PRD **82**, 101301R (2010)

- Mass-Radius relation from bursts
- Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
  2. 3 transient low mass X-ray binaries
  3. 1 isolated cooling NS, RX J1856-3754

Isospin asymmetric matter

Tuning correlations

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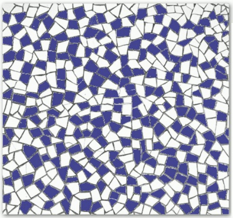
Nuclear "trencadís"

β=0

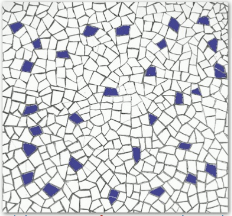
Symmetric matter



SR+Tensor correlations

β≠0

Asymmetric matter

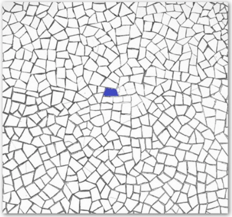


*Neutrons **less** correlated*

*Protons **more** correlated*

β≈1

Polaron



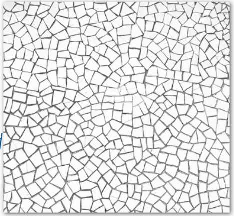
*Protons **maximally** correlated*

***Hyper**-impurities?*

Neutron stars

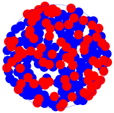
β=1

Neutron matter



SR correlations

Nuclei



$$\beta = \frac{N-Z}{N+Z}$$

Frick, Rios et al. PRC **71**, 014313 (2005)
Rios et al. PRC **79**, 064308 (2009)
Carbone et al. EPL **97** 22001 (2012)

Effective approach to the EoS

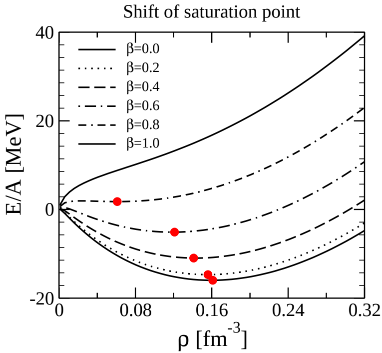
- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \Longleftrightarrow \frac{E}{A}(\rho, \beta) = ?$$

$$\beta = \frac{N - Z}{N + Z}$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = 0$
  - Isospin symmetry $\Rightarrow$ even powers of β
  - Give the coefficients a name!

$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho_0, \beta) + 3\rho_0 \frac{\partial E/A}{\partial \rho}\Big|_{\rho_0} \left(\frac{\rho - \rho_0}{3\rho_0}\right) + \frac{9\rho_0^2}{2!} \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho_0} \left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(3)$$



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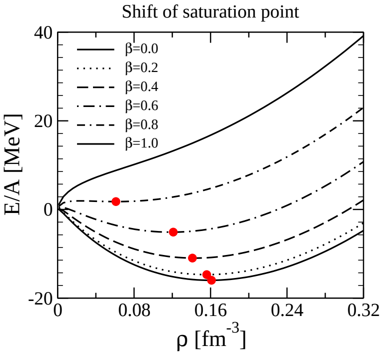

$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho_0, 0) + \frac{1}{2!} \frac{\partial^2 E/A}{\partial \beta^2}\Big|_{\rho_0, \beta=0} \beta^2 + \frac{3\rho_0}{2!} \frac{\partial^3 E/A}{\partial \beta^2 \partial \rho}\Big|_{\rho_0, \beta=0} \beta^2 \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{9\rho_0^2}{2!} \left\{ \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho_0, \beta=0} + \frac{1}{2!} \frac{\partial^4 E/A}{\partial \rho^2 \beta^2}\Big|_{\rho_0, \beta=0} \beta^2 \right\} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \mathcal{O}(3, 2)$$

Effective approach to the EoS

- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \iff \frac{E}{A}(\rho, \beta) = ?$$

$$\beta = \frac{N - Z}{N + Z}$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = 0$
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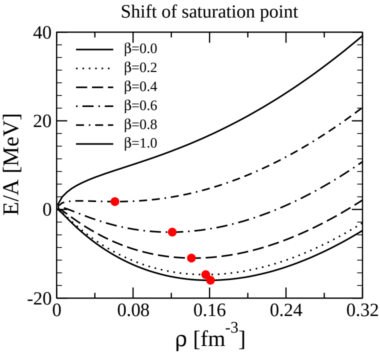
  - Give the coefficients a name!

$$\frac{E}{A}(\rho, \beta) = E_0 + E_{sym}\beta^2 + L\beta^2 \left(\frac{\rho - \rho_0}{3\rho_0}\right) + \frac{1}{2!}\left\{K_0 + K_{sym}\beta^2\right\}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(3,2)$$



EoS from basic nuclear properties

An incomplete list

$$\frac{E}{A}(\rho,\beta) = E_0 + E_{sym}\beta^2 + L\beta^2\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2!}\left\{K_0 + K_{sym}\beta^2\right\}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

Quantity	Experimental probes	Value	Ref.
ρ_0	(e,e') elastic scattering	$0.16\ \mathrm{fm}^{-3}$	[1]
E_0	$\frac{E}{A}$ bulk systematics	-16 MeV	[1]
K_0	GMR energy in $Z\sim N$	240 ± 20 MeV	[2]
E_{sym}	$\frac{E}{A}$ bulk systematics + ID	32 ± 2 MeV	[3]
L	ID, IVMR energies, δR	61 ± 11 MeV	[3]
K_{sym}	?	?	

[1] Schuck & Ring, *The Nuclear Many-Body Problem* (Springer)
[2] Blaizot, Phys. Reps. 64, 171 (1981)
[3] Tsang *et al.*, Phys. Rev. Lett. 102, 122701 (2009)

Symmetry energy & nuclear structure

$^{208}\mathrm{Pb}$ skin thickness & L

Neutron-matter pressure is dominated by L:

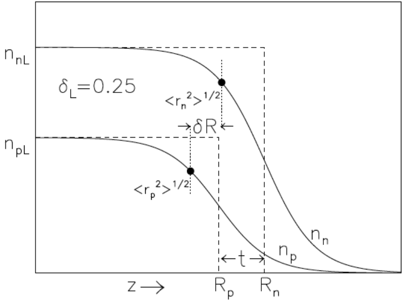
$$p(\rho_0, \beta) = \frac{\rho_0 \beta^2}{3} L$$

Does it correlate with nuclear observables?

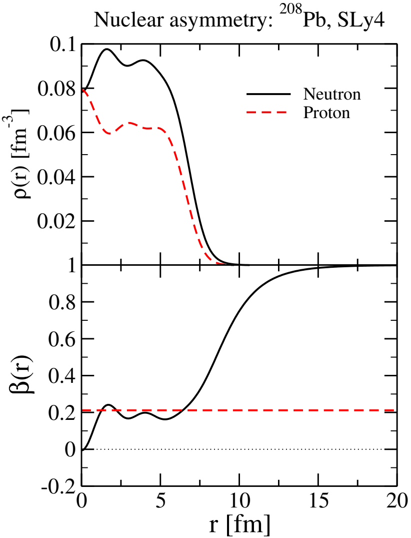
Symmetry energy & nuclear structure

^{208}Pb skin thickness: $\delta R = R_n - R_p$

Neutron skin sketch

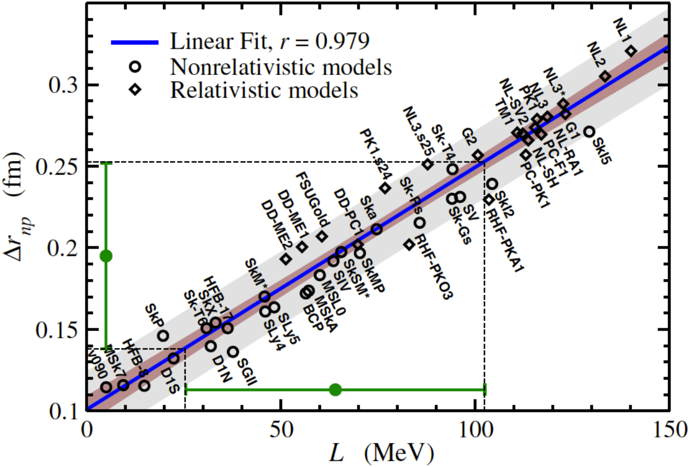


A. Steiner *et al.*, Phys. Rep. **411**, 325 (2005)



Symmetry energy & nuclear structure

δR vs L



S. Typel & B. Alex Brown, PRC **64**, 027302 (2001)

X. Roca-Maza *et al.*, PRL **106**, 252501 (2011)

Symmetry energy & nuclear structure

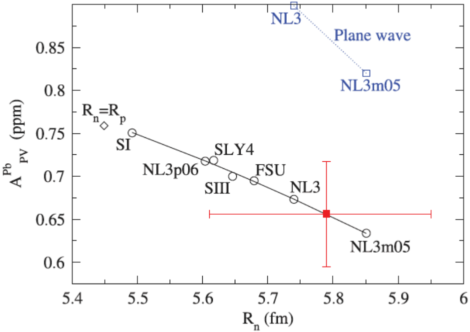
Parity violating electron scattering

$$q_n^W = -1$$
$$q_p^W = 1 - 4sin^2\Theta_W \sim 0.05$$
$$A_{\text{PM}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

Free of strong interaction uncertainties.

C. Horowitz *et al.*, PRC **63**, 025501 (2001)
C. Horowitz *et al.*, PRC **85**, 032501(R) (2012)

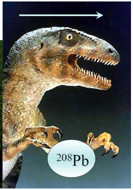
$\delta R = 0.33 \pm 0.17$ fm

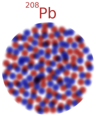


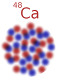
Abrahamyan *et al.*, PRL **108**, 112502 (2012)

Lead Radius Experiment
PREX
(early 2010 in Hall A)

Jefferson Lab





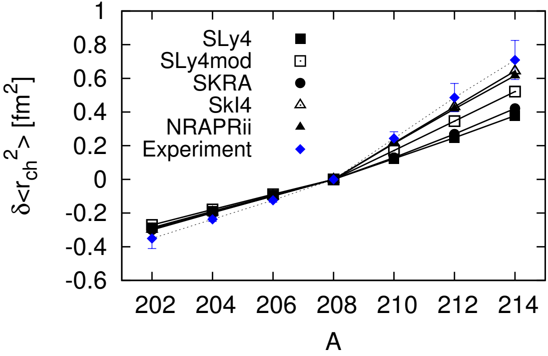


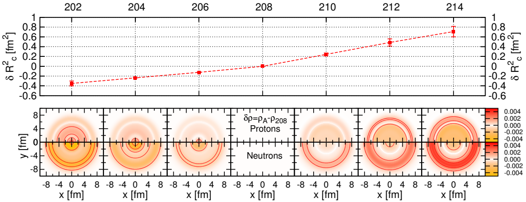
PREX CREX

Edinburgh exp.: π^0 photoproduction

Kink in charge raddi

Radii of lead isotopes: open questions

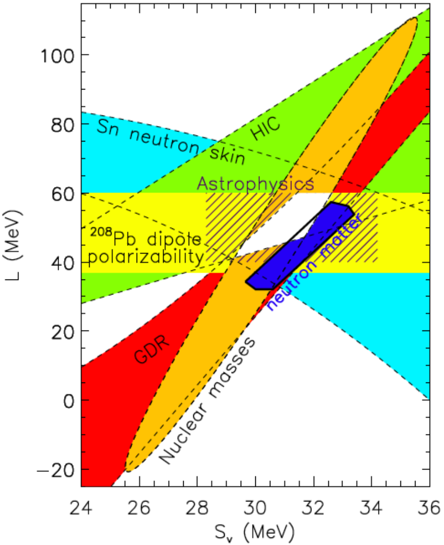




Goddard, Stevenson & Rios, PRL **110**, 032503 (2013)

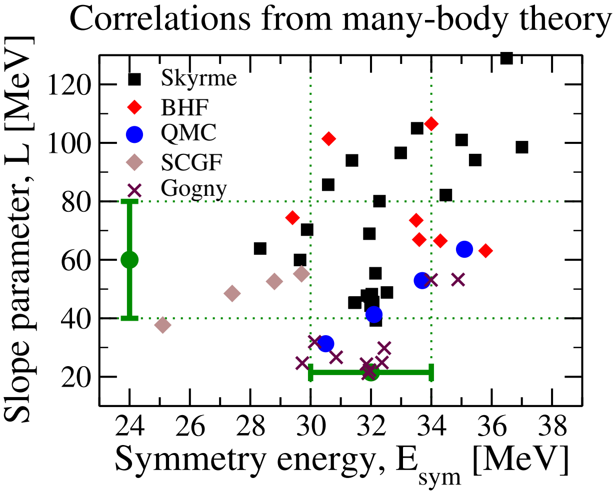
Correlation between observables

Correlations from different observables



Lattimer & Lim, ApJ **771**, 51 (2013), arXiv:1203.4286.

Correlation between observables



Gogny: Rosh Sellahewa, PhD

Ab initio nuclear theory

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Many-body technique

✦BHF, FHNC, AFDMC, SCGF

Empirical properties

✦Saturation density
✦Binding energy
✦Compressibility

Hamiltonian

✦**Phase-shift equivalent NN force**
✦NNN force?

Degrees of freedom

✦**Nucleons**
✦Hyperons, quarks, mesons?

Framework

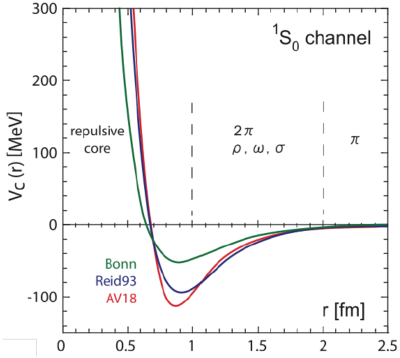
✦**Non-relativistic & quantum**
✦Relativistic, classical?

Complications

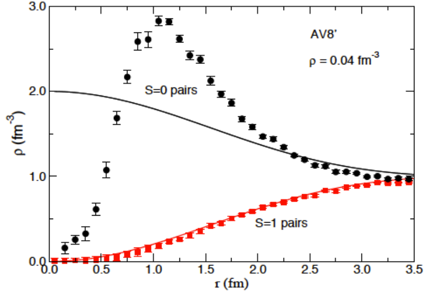
The hard life of nuclear many-body physicists

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

Different NN potentials



GFMC pair distribution function

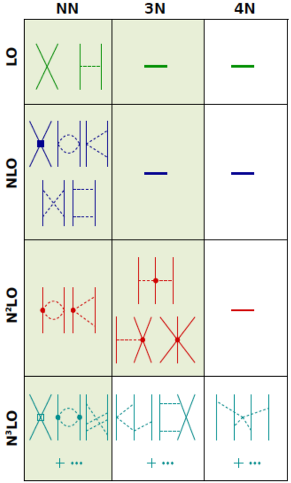


Carlson *et al.*, Phys.Rev. C **68**, 025802 (2003)

- NN interaction is not uniquely defined
- Short-range core needs many-body treatment
- Chiral expansion ⇒ systematics & many-body forces

Complications

The hard life of nuclear many-body physicists

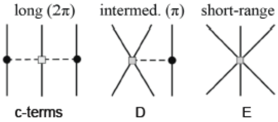


Robert Roth – TU Darmstadt – 04/2013

Chiral perturbation theory

1. π and N as dof
2. Systematic expansion in diagrams
3. 2N force at N^3LO - LEC are fitted
4. 3N force at N^2LO - 2 more LECs
5. 4N force are small

Tews, Schwenk *et al.*, PRL **110** 032504 (2013)



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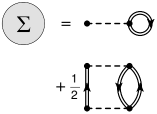
3BFs in Green´s functions theory

Diagrammatic expansion with 3BF

Self-energy expansion to 2nd order



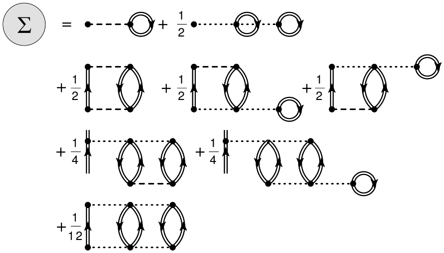
- Only skeleton 1PI diagrams needed

Effective interaction expansion

3BFs in Green´s functions theory

Diagrammatic expansion with 3BF

Self-energy expansion to 2nd order

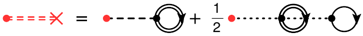


- Only skeleton 1PI diagrams needed
- Number of diagrams substantially increases
- Usable in higher order resummations

Effective interaction expansion

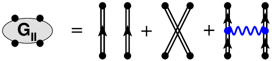
Define effective 1B and 2B forces

Effective one-body force

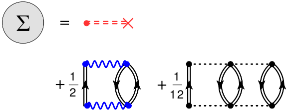


Effective two-body force

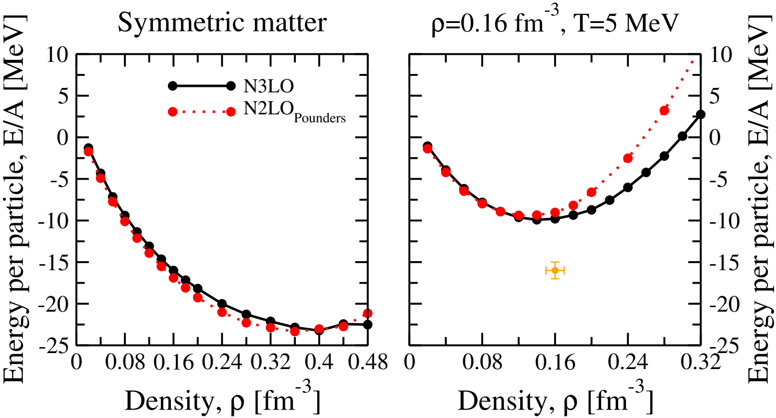
Two-body propagator



Rewrite self-energy expansion



Ab initio EoS from chiral interactions

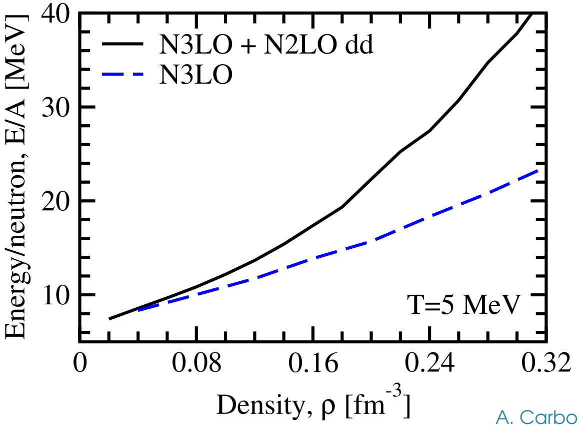


A. Carbone, A. Rios & A. Polls

- Saturation properties of N3LO and NNLO similar
- NNLO ⇒ 2B & 3B same order in χ expansion
- Finite temperature & asymmetry also accessible

Ab initio EoS from chiral interactions

Pure neutron matter EoS



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Ab initio EoS from chiral interactions

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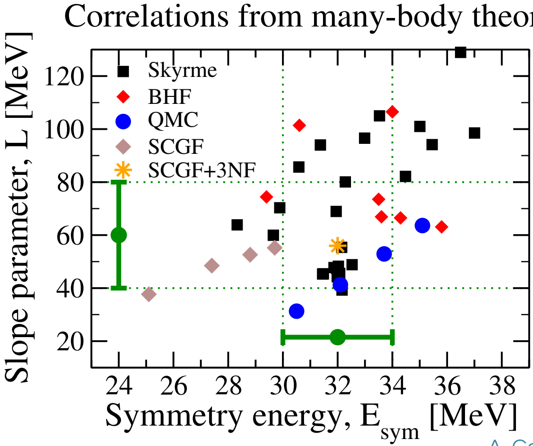

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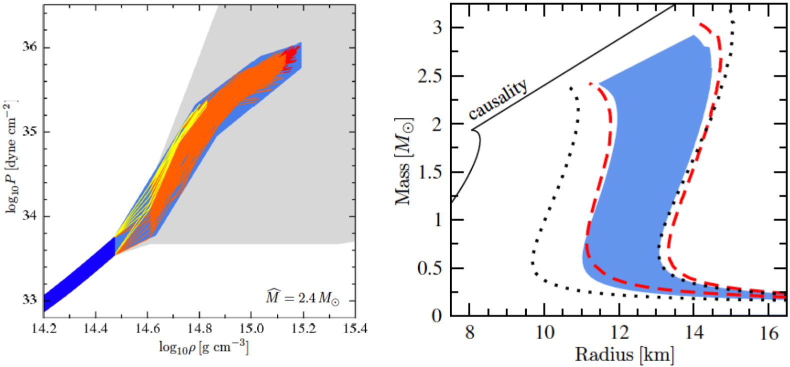
Ab initio EoS from chiral interactions

Impact on $M - R$ relation: causality + $2.4 M_\odot$



Hebeler, Lattimer, Pethick & Schwenk, arxiv:1303.4662

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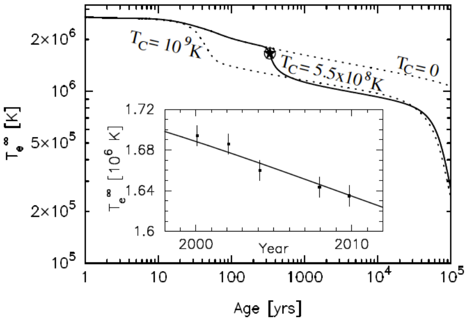
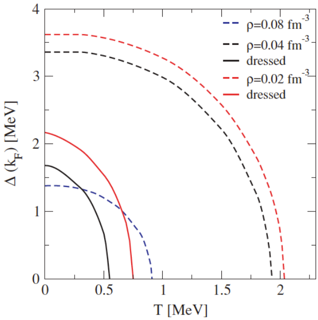
Pairing properties

Cooling of Cassiopea A & $^3PF_2$ pairing



Page *et al.*, PRL **106**, 081101 (2011)

CDBonn: $^1S_0$ pairing gap



Muther & Dickhoff, PRC **72** 054313 (2005)

$$\Delta_{lk}^{JST} = -\sum_{l'} \int_0^\infty \mathrm{d}k' k'^2 \langle kl | V^{JST} | k'l' \rangle \frac{\Delta_{l'k'}^{JST}}{2\chi_{k'}}$$

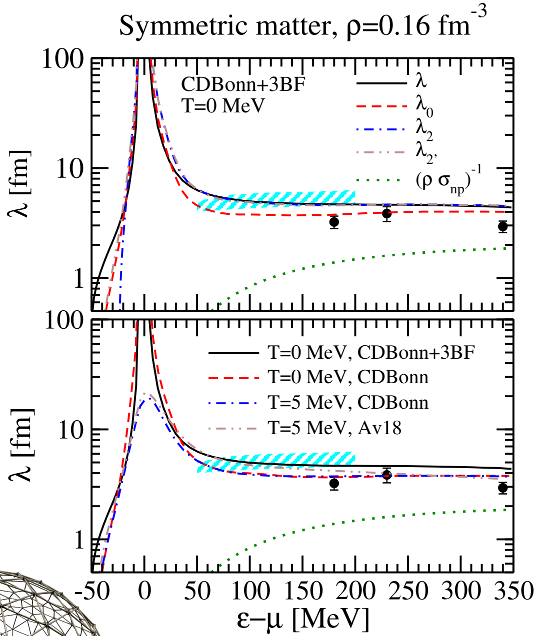
BCS / quasi-particle

$$\frac{1}{2\chi_k} = \frac{1 - 2f(\varepsilon_k)}{2\varepsilon_k}$$

Beyond quasi-particle

$$\frac{1}{2\chi_k} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega'}{2\pi} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A_k(\omega) A_k^s(\omega')$$

Transport: nucleon mean-free path



$$\lambda_k = \frac{1}{\Gamma_k} \frac{\partial \varepsilon_k}{\partial k}$$

- $\lambda \sim 4-5$ fm above 50 MeV
- Compatible with pA experiments
- Small model dependence
  - $\lambda_0 \Rightarrow$ no non-locality
  - $\lambda_2 \Rightarrow$ full non-locality
  - $\lambda_2' \Rightarrow m_k^*$ non-locality
- Classical approximation is incorrect!
- Little effect of 3BFs

A. Rios & V. Somà, PRL **108**, 012501 (2012)

Conclusions

- Ab initio description of nuclear & neutron matter
- Fully self-consistent & quantum mechanical calculation

- Single-particle microscopic properties ✔
- Thermodynamic properties ✔
- Mean-free path in dense matter ✔

- Adding three-body forces consistently
- Pairing: $^1S_0$, $^3PF_2$
- Other transport properties are coming

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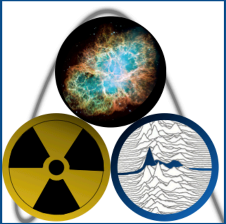
Thanks!



Neutron Stars

Nuclear Physics, Gravitational Waves & Astronomy

29-30 July 2013

Institute of Advanced Studies, University of Surrey
http://www.ias.surrey.ac.uk/workshops/neutstar/

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